

Effective Masses and Sizes of $N(939)$, $\Delta(1232)$ and $N(1440)$ in Nuclear Medium

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Abstract

The effective masses and sizes of $N(939)$, $\Delta(1232)$ and $N(1440)$ in nuclear medium are calculated for a model in which quarks are subjected to a confinement force quenched by the scalar part of nuclear interactions. At the nuclear matter density, the nucleon swells by 50% relative to the free nucleon. The masses of $\Delta(1232)$ and $N(1440)$ decrease in nuclear medium more than the nucleon, so that the pion requires less energy to excite them in nuclear medium than it does for a free nucleon.

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The mass of the nucleon has its dynamical origin in the fundamental dynamics of hadrons, quantum chromodynamics (QCD). The most important ingredient of QCD for the nucleon mass is the confinement force for three quarks in the nucleon. When a nucleon is brought into nuclear medium, it is exposed to complex interactions with other nucleons. A part of the interaction which transforms as a Lorentz scalar can be absorbed into the mass of the nucleon to define the effective mass. In this paper we try to describe the change from the free nucleon mass to the effective mass by quenching the confinement strength due to the scalar part of the nuclear interaction.

As a consequence of this quenching, a nucleon swells in nuclear medium. The notion of nucleon swelling has received much attention in connection to the interpretation of the earlier EMC experiments, [1] and the Okamoto-Nolen-Schiffer anomaly. [2] The recent experiments of the low energy scattering of K^+ on some nuclei suggest that the nucleon swelling actually takes place within nuclei. [3] The simple picture provided by our model also enables us to discuss the effective masses of other baryons such as $\Delta(1232)$ and $N(1440)$ in nuclear medium. The purpose of this paper is thus twofold: to investigate both nucleon swelling and the effective masses of $\Delta(1232)$ and $N(1440)$ in nuclear medium.

We use the bag model whose surface has a dynamical degree of freedom with the radius R and its conjugate momentum P_b . The model was proposed by Brown, Durso and Johnson, [4] and by Tomio and Nogami (TN). [5,6] The Hamiltonian describing SU(2) baryons, of which we are interested in $N(939)$, $\Delta(1232)$ and $N(1440)$ (denoted simply as N , Δ and R hereafter), is given by

$$H = \sum_{i=1}^3 \{ \boldsymbol{\alpha}_i \cdot \mathbf{p}_i + \beta_i V(R, r_i) \} + \frac{g_s}{R} \sum_{i>j} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + H_b, \quad (1)$$

where $V(R, r_i)$ is zero for $r_i < R$ and infinity for $r_i > R$. We added the approximate hyperfine interaction with the coupling constant g_s to the usual bag Hamiltonian to remove the degeneracy between two spin-partners of the SU(2) baryons such as N and Δ . [7] The Hamiltonian to describe the dynamical motion of the bag is

$$H_b = \frac{P_b^2}{2M_B} + \frac{4\pi}{3} B_D R^3 - \frac{Z_0}{R} + U_\pi(R) \quad (2)$$

with an *ad hoc* inertial mass M_B which is assumed to be proportional to the mass of N or Δ with a proportional constant β . [4] The term U_π in Eq. (2) is the self-energy due to the π -quark interaction, containing the intermediate states restricted to N or Δ . We use the U_π of TN with a little modification:

$$U_\pi = -3 \frac{f_N^2}{\pi m_\pi^2} \int_0^\infty dk \frac{k^4 v^2(k, R)}{\omega_k(\omega_k + E_N - m_N)} - \frac{96}{25} \frac{f_N^2}{\pi m_\pi^2} \int_0^\infty dk \frac{k^4 v^2(k, R)}{\omega_k(\omega_k + E_\Delta - m_\Delta)} \quad (3)$$

for N and R, and

$$U_\pi = -3 \frac{f_N^2}{\pi m_\pi^2} \int_0^\infty dk \frac{k^4 v^2(k, R)}{\omega_k(\omega_k + E_\Delta - m_\Delta)} - \frac{24}{25} \frac{f_N^2}{\pi m_\pi^2} P \int_0^\infty dk \frac{k^4 v^2(k, R)}{\omega_k(\omega_k + E_N - m_\Delta)} \quad (4)$$

for Δ , where f_N is the π N axial-vector coupling constant, m_π the pion mass, $\omega_k = (m_\pi^2 + k^2)^{1/2}$, and $E_B = (m_B^2 + k^2)^{1/2}$ for B=N and Δ . The form factor is defined by

$$v(k, R) = \frac{3j_1(kR)}{kR} \exp \left[-\frac{k^2 R_\pi^2}{6} \right], \quad (5)$$

where R_π is a phenomenological parameter related to pion radius. Note that we ignore the isospin degree of freedom so that the proton and the neutron are degenerate.

We apply the adiabatic approximation to separate the bag dynamics from quark motion. [5] In this scheme one estimates the quark energy by ignoring the bag motion; the quark energy thus obtained is a function of the bag radius R and is regarded as the potential for the bag motion. Then N and Δ , with $S = 1/2$ and $3/2$, emerge as the states with the lowest eigenvalues of the equation describing the bag motion:

$$\left[H_b + \frac{3\eta_0}{R} \mp \frac{3g_s}{R} \right] b_B(R) = m_B b_B(R), \quad (6)$$

where the lower suffix B stands for N or Δ , $\eta_0 = 2.04$, and the upper and lower signs correspond to N and Δ , respectively. In this model the R is interpreted to be the first breathing excitation of the nucleon.

The model contains six parameters of which f_N^2 is fixed to be 0.08. We also use the MIT value for B_D : $B_D^{1/4} = 0.145 \text{ GeV}$. [7] The pion charge radius was measured to be around 0.66fm. However this should not be identified with the spatial extension of quark components

which we regard as R_π . Several estimates put R_π between 0.33fm and 0.40fm. [8] We regard any R_π in this interval as the experimental value of the root-mean-square (rms) radius of the pion. The rest of the parameters are determined to fit the observed masses of N, Δ and R, and are $\beta = 0.516$, $Z_0 = 2.201$ and $g_s = 0.226$, which yield $R_\pi = 0.38\text{fm}$.

Let us turn to baryons in nuclear medium. We attempt to replace complex nuclear interactions on a baryon by the quenched confinement force for three quarks in the baryon. It is specified in such a way that Eq. (6) for the nucleon gives the effective mass obtained from conventional nuclear physics. We use the standard $\sigma\omega$ model, referring to [9] for the details. The effective mass of the nucleon, m_N^* , is defined by

$$m_N^* = m_N - g_\sigma \sigma \quad (7)$$

in the mean-field approximation for σ . Quantities appearing with an asterisk hereafter are to be understood as the quantities in nuclear medium. The mean-field σ satisfies the following equations:

$$g_\sigma \sigma = \frac{g_\sigma^2}{m_\sigma^2} [\rho_S - b_\sigma m_N (g_\sigma \sigma)^2 - c_\sigma (g_\sigma \sigma)^3] , \quad (8)$$

$$\rho_S = \frac{4}{(2\pi)^3} \int d^3k \frac{m_N^*}{\sqrt{k^2 + m_N^{*2}}} \theta(k_F - k) , \quad (9)$$

where we consider our system as nuclear matter filled by nucleons up to the level of k_F . We use $g_\sigma^2 = 154.050$, $b_\sigma = 0.000673$, $c_\sigma = 0.009786$ and $m_\sigma = 550\text{MeV}$, which were obtained by Gmuca to reproduce the Dirac-Bruckner-Hartree-Fock results of nuclear matter calculation. [10] Two typical values of the effective mass parameter defined by $\alpha_N = m_N^*/m_N$ are 0.691 and 0.619 for $k_F = 1.20$ and 1.36 fm^{-1} , respectively. The value of $k_F = 1.36 \text{ fm}^{-1}$ is for the normal nuclear matter and 1.20 fm^{-1} is to simulate medium heavy nuclei.

Now we regard the change of m_B to m_B^* as the consequence of quenching of the parameters in our quark model due to nuclear interactions. We keep β , Z_0 and g_s , which are related to the intrinsic nature of the bag interior, constant throughout. Thus f_N , m_π , R_π and B_D are left as the parameters to be quenched by nuclear interactions.

The effect of nuclear interactions on f_N was studied by Rho under the assumption of the relation $g_N/g_A = g_N^*/g_A^*$, where g_N and g_A are the πN pseudo-scalar and the axial-vector coupling constants, respectively. [11] The equivalence theorem connecting g_N to f_N derives

$$\frac{(f_N/m_\pi)^*}{(f_N/m_\pi)} = \frac{1}{\alpha_N} \frac{g_A^*}{g_A}. \quad (10)$$

We quote [11] for the calculation of g_A^* .

We know the properties of pions in nuclear medium as little as those of the free pion. The particular properties of pions are associated with the chiral symmetry of the strong interaction and are beyond the simple bag-based description. Let us use the following conventional model to describe the pion. The effective Hamiltonian is given by

$$H_M = \sum_{i=1}^2 \{ \boldsymbol{\alpha}_i \cdot \mathbf{p}_i + \beta_i V(R, r_i) \} + 2 \frac{g_s}{R} \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \frac{4\pi}{3} B_D R^3 - \frac{Z_\pi}{R} + H_{q\bar{q}}, \quad (11)$$

where Z_π is an adjustable parameter corresponding to Z_0 for baryons. We introduced the phenomenological $q\bar{q}$ interaction, $H_{q\bar{q}}$. It is needed to bring m_π down to the observed value, otherwise m_π comes out to be too large. Presumably q and \bar{q} will interact strongly when they are at the largest separation inside the bag. Considering this, we take $H_{q\bar{q}}$ to be [12]

$$H_{q\bar{q}} = -\lambda^2 R^3 \delta(r_1 - R) \delta^{(3)}(\mathbf{r}_1 + \mathbf{r}_2). \quad (12)$$

We ignored the bag motion here for simplicity so that the pion emerges as the ground state with the lowest eigenvalue minimized with respect to R .

In order to solve the eigenvalue equation for H_M , we diagonalize it within the basis of the eigenstates of the Hamiltonian without $H_{q\bar{q}}$. Since we are interested in the ground state, it will be sufficient to truncate the diagonalization space with the first three states ($1S_{1/2}$, $1P_{3/2}$, $1P_{1/2}$) which accommodate the pair of q and \bar{q} . Two parameters, $Z_\pi = 2.257$ and $\lambda = 0.256$, were determined by fitting $m_\pi = 138\text{MeV}$ and $R_\pi = 0.38\text{fm}$. We assume that these parameters are out of the influence of nuclear interactions for the same reason as in the baryon case. Hence only the quenched B_D determines m_π^* and R_π^* in nuclear medium. The quenched B_D for the pion may be different from that for the nucleon. However, our

inadequate knowledge of pion properties in nuclear medium leads us to a common B_D^* for both the nucleon and pion.

We determine B_D^* according to the following procedure. The Hamiltonian describing baryons in nuclear medium depends on m_N^* , m_Δ^* and B_D^* through f_N^* , M_B^* and U_π in which we replace $1/(\omega_k + E_N - m_B)$ by $\theta(k - k_F)/(\omega_k^* + E_N^* - m_B^*)$ with B=N in Eq. (3) and with B= Δ in Eq. (4) to take account of the Pauli principle. The lowest eigenvalues of Eq. (6) are m_N^* for $S = 1/2$ and m_Δ^* for $S = 3/2$. Thus the eigenvalue equation is a set of coupled-selfconsistent equations for m_N^* and m_Δ^* , and hence for α_N and α_Δ . One can determine B_D^* such that the selfconsistency is satisfied for a given α_N . Once B_D^* is fixed, the effective masses of excited baryons can be evaluated by solving the eigenvalue equation (6). The rms radius of a baryon B is calculated according to

$$\langle r^2 \rangle_{B(q)}^{1/2} = 0.73 \left[\int_0^\infty dR b_B^*(R) R^2 b_B(R) \right]^{1/2}. \quad (13)$$

The effective masses and the nucleon rms radius thus obtained are shown in Figs.1 and 2, respectively. Obviously the nucleon swells in nuclear medium as nucleon density is increased and the rms radius reaches $[\langle r^2 \rangle_{N(q)}^*]^{1/2} / [\langle r^2 \rangle_{N(q)}]^{1/2} = 1.96$ at the nuclear matter density. This large number is not particularly surprising. The rms radius evaluated by Eq. (13) is of the quark core. Consider that the physical nucleon consists of the three-quark core and a pion. Then the rms radius is given by [6]

$$\langle r^2 \rangle_N = P_{3q} \langle r^2 \rangle_{N(q)} + (1 - P_{3q}) \langle r^2 \rangle_{N(\pi)}, \quad (14)$$

where P_{3q} is the probability to find the nucleon in the three-quark state and is given by

$$P_{3q} = \left[1 + \frac{3f_N^2}{\pi m_\pi^2} \int_0^\infty dk \frac{k^4 \bar{v}^2(k)}{\omega_k(\omega_k + E_N - m_N)^2} + \frac{96f_N^2}{25\pi m_\pi^2} \int_0^\infty dk \frac{k^4 \bar{v}^2(k)}{\omega_k(\omega_k + E_\Delta - m_N)^2} \right]^{-1} \quad (15)$$

with $\bar{v}^2(k) = \int_0^\infty dR b_N^*(R) v^2(k, R) b_N(R)$. The second term in Eq. (14) is the pion contribution. It should be understood, when we calculate P_{3q} in nuclear medium, we replace the

quantities in Eq. (15) by the corresponding ones with an asterisk and make the similar replacement made in U_π to account for the Pauli principle. Our calculation gives $P_{3q} = 0.74, 0.87$ and 0.91 for $k_F = 0.00, 1.20$ and 1.36fm^{-1} , respectively. The measured value for a free nucleon, $\langle r^2 \rangle_N^{\text{exp}} = (0.86 \text{ fm})^2$, gives $\langle r^2 \rangle_{N(\pi)} = (1.27 \text{ fm})^2$. Let us use this value in nuclear medium, anticipating that the pionic contribution becomes less significant as the density increases. The result is shown by the dashed line in Fig.2. The typical values are $\langle r^2 \rangle_N^{1/2} = 1.13$ and 1.29fm for $k_F = 1.20$ and 1.36fm^{-1} respectively. This tells us that the nucleon radius swells by 50% at the nuclear matter density. This number is close to the 45% obtained in the skyrmion picture. [11] Can one estimate the nucleon density n_c at which a swollen bag comes into contact with another so that quarks in the bag may squeeze into the other? Let us define n_c such that the volume of the quark core swells to occupy the mean volume shared by a nucleon at n_c , hence $\langle r^2 \rangle_{N(q)}^{1/2} = (9\pi/8)^{1/3}/k_F$ at n_c . This condition is reached at $k_F = 1.28\text{fm}^{-1}$ which corresponds to 0.8 times the normal nuclear matter density.

In regard to the effective masses, Δ reduces its mass more than the nucleon. Consequently the pion requires less energy to excite the nucleon to the Δ in nuclear medium than it does outside. The mass difference between Δ and N decreases to 50% of that for free particles at the nuclear matter density. Our number is much larger than the 20% predicted in the Nambu-Jona-Lasinio model. [14] In our definition of the effective mass only the interaction which transforms as a Lorentz scalar is taken into account, so that the nucleons with the effective mass are still interacting with each other through the vector field. The vector interaction may work in a different way for N and Δ , and must be taken into account for quantitative discussion of the downward shift of the Δ resonance observed in π -nucleus scattering. [13] Our result also predicts that R will be excited with less energy in π -nucleus scattering than in π N scattering. This is signalled by the large inelasticity in the two-pion channel at lower pion energy than in π N scattering.

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Figure Captions

Fig.1

The effective masses of $N(939)$ (the solid line), $\Delta(1232)$ (the dashed line) and $N(1440)$ (the dot-dashed line). Also the mass difference, $\delta m = m_{\Delta}^* - m_N^*$, is shown by the dotted line. The arrow indicates the Fermi momentum corresponding to the nuclear matter density.

Fig.2

The root-mean-square radii of the nucleon in nuclear medium. The solid line represents the rms radii of the three-quark core in the nucleon, and the dashed line those of the physical nucleon estimated by using Eq. (14). The arrow indicates the Fermi momentum corresponding to the nuclear matter density.